

NAG Toolbox for MATLAB

f07bu

1 Purpose

f07bu estimates the condition number of a complex band matrix A , where A has been factorized by f07br.

2 Syntax

```
[rcond, info] = f07bu(norm_p, kl, ku, ab, ipiv, anorm, 'n', n)
```

3 Description

f07bu estimates the condition number of a complex band matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^H)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – string

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'

$\kappa_1(A)$ is estimated.

norm_p = 'I'

$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **kl** – int32 scalar

k_l , the number of subdiagonals within the band of the matrix A .

Constraint: **kl** ≥ 0 .

3: **ku** – int32 scalar

k_u , the number of superdiagonals within the band of the matrix A .

Constraint: **ku** ≥ 0 .

4: **ab(ldab,*)** – complex array

The first dimension of the array **ab** must be at least $2 \times \mathbf{kl} + \mathbf{ku} + 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The *LU* factorization of *A*, as returned by f07br.

5: **ipiv(*)** – int32 array

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

The pivot indices, as returned by f07br.

6: **anorm** – double scalar

If **norm_p** = '1' or 'O', the 1-norm of the **original** matrix *A*.

If **norm_p** = 'I', the ∞ -norm of the **original** matrix *A*.

anorm must be computed either **before** calling f07br or else from a **copy** of the original matrix *A*.

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters1: **n** – int32 scalar

Default: The second dimension of the array **ab**.

n, the order of the matrix *A*.

Constraint: **n** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, work, rwork

5.4 Output Parameters1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of *A*. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, *A* is singular to working precision.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter *i* had an illegal value on entry. The parameters are numbered as follows:

1: **norm_p**, 2: **n**, 3: **kl**, 4: **ku**, 5: **ab**, 6: **ldab**, 7: **ipiv**, 8: **anorm**, 9: **rcond**, 10: **work**, 11: **rwork**, 12: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07bu involves solving a number of systems of linear equations of the form $Ax = b$ or $A^H x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8n(2k_l + k_u)$ real floating-point operations (assuming $n \gg k_l$ and $n \gg k_u$) but takes considerably longer than a call to f07bs with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The real analogue of this function is f07bg.

9 Example

```

norm_p = '1';
m = int32(4);
kl = int32(1);
ku = int32(2);
ab = [complex(0, 0), complex(0, 0), complex(0, 0), complex(0, 0);
      complex(0, 0), complex(0, 0), complex(0.97, -2.84), complex(0.59, -
0.48);
      complex(0, 0), complex(-2.05, -0.85), complex(-3.99, +4.01),
complex(3.33, -1.04);
      complex(-1.65, +2.26), complex(-1.48, -1.75), complex(-1.06, +1.94),
complex(-0.46, -1.72);
      complex(0, +6.3), complex(-0.77, +2.83), complex(4.48, -1.09),
complex(0, 0)];
anorm = 15.47935040206279;
[ab, ipiv, info] = f07br(m, kl, ku, ab);
[rcond, info] = f07bu(norm_p, kl, ku, ab, ipiv, anorm)

rcond =
    0.0096
info =
    0

```